Lecture 1 — Possible Worlds: Motivations and Applications

Admin: Lectures run 12–1pm, weeks 1–4, last 50 minutes, with 5 minutes for questions. My email address is james.openshaw@philosophy.ox.ac.uk.

Lecture 1: Introduction to the notion of possible worlds. An informal account of possible worlds semantics (PWS) in modal logic and how this has sustained an interest in modality. What the intended explanatory work of possible worlds (PWs) is.

Lecture 2: We’ll start with further discussion of the theoretical applications of possible worlds. Then we’ll explore the most systematic and infamous realist account in detail: Lewis’ genuine modal realism.

Lecture 3: Ersatz realism/abstractionism. Is it possible to reap the fruits of PWs and PWS without being committed to the claim that possible worlds are the same kind of thing as the actual world?

Lecture 4: Anti-realism. What anti-realism might consist in generally and why it might be a worthy option in the case of PWs. We will look at two of the more plausible and developed accounts: fictionalism and quasi-realism.

1. Introduction

1.1 Possibilities and Possible Worlds

What possible worlds are not. The possible worlds of philosophy are not ‘parallel universes’ or ‘alternate realities’ of the sort familiar from sci-fi or the ‘many worlds’ of Quantum Mechanics.

Disanalogies. Only on certain extreme views of PWs are they the same kind of thing as the actual world. Second, the latter invariably have parts which engage in causal relations, so they couldn’t be properly distinct possibilities. Finally, QM’s ‘many worlds’ exist contingently—something many philosophers will reject concerning possible worlds.

I believe, and so do you, that [...] there are many ways things could have been besides the way they actually are. On the face of it, this sentence is an existential quantification. [...] I therefore believe in the existence of entities that might be called ‘ways things could have been’. I prefer to call them possible worlds (Lewis 1973: 84).

The notion of possibilities infects a lot of our thinking and theorizing. Would a complete theory of everything that exists and each thing’s categorical properties miss anything out?

On the face of it, yes. It’s important in science to figure out how things would go under such and such circumstances. For example, even if no glass were ever broken, that wouldn’t mean wine glasses were not fragile.
We also have conditional knowledge expressed by the following sort of statement:

\[(1) \quad \text{If the patient hadn’t recovered after we administered chloroquine, we’d have known the strain of malaria was (the chloroquine-resistant) } P. \text{ falciparum.}\]

Counterfactual thought, and thought about ways things might have been, is not idle daydreaming, then. This sort of thinking is inseparable from our ordinary causal thinking about the natural world. To know A caused B is some way to knowing that if A hadn’t happened, B wouldn’t have. We’ll look more at counterfactuals later...

Varieties of (im)possibility:

- You couldn’t get to Ho Chi Minh City by tomorrow evening (GMT). \quad \text{Practical}
- Nothing can travel faster than light. \quad \text{Natural}
- The number of people in this room couldn’t be both even and not even. \quad \text{Absolute}
- You can’t drink alcohol in the UK if you’re under 18. \quad \text{Legal}
- It can’t be 1pm already. \quad \text{Doxastic}
- You can’t buy eggs from caged hens. \quad \text{Deontic}

We can define modalities of different kinds in terms of what is the case at a specified range of worlds: those compatible with some set of constraints (the strictures of what is practical, the laws of nature, the laws of metaphysics...).

It is controversial whether we can identify the above totality of worlds with those conforming to some particular set of constraints (e.g. metaphysical laws, or logical laws...). But whether it is subject to any more informative characterisation (or is fundamental or perfectly natural), the modality so-captured is (i) alethic and (ii) absolute.

Alethic modalities are those for which the actual world is always possible. So the following inference is valid.

\[
\begin{align*}
\text{Necessarily } p. \\
\text{Actually } p.
\end{align*}
\]

A modality \( m \) is \textit{absolute} iff all and only the full range of \textit{genuinely possible} worlds are \( m \)-possible.

1.2 Semantics: extensions and intensions

Traditionally, the intension of a predicate is a \textit{property}, the extension of a predicate is a \textit{set}. The predicates ‘is a featherless biped (that is not a plucked chicken)’ and ‘is human’ share their extension (suppose) but have different intensions.
Basic extensional semantics:
- The extension of a singular term \( \alpha \) = the object designated by \( \alpha \).
- The extension of an \( n \)-place predicate \( F^n \) = the set of \( n \)-tuples of objects which fall under the predicate \( F^n \).
- The extension of a sentence = its truth-value.
- A sentence of the form \( Fa \) is true iff the extension of \( a \) is in the extension of \( F \).

**Intensional** constructions are those that distinguish among fillings with equivalent extensions. Compare ‘Necessarily, all bachelors are unmarried’ with ‘Necessarily, John is unmarried’.

**Extensionality:** Suppose \( \varphi \) and \( \Psi \) are terms of language \( L \), and that \( \varphi \) contains \( \Psi \). \( L \) is extensional iff, whenever \( \Psi^* \) has the same extension as \( \Psi \), and whenever \( \varphi^* \) is the result of replacing all occurrences of \( \Psi \) with \( \Psi^* \), the extension of \( \varphi \) is equivalent to that of \( \varphi^* \).

In contrast to the clarity achieved by extensional semantics in the early 20\(^{th}\) Century, no similar consensus about how to formally characterise the meanings of constructions like ‘Necessarily, \( S \)’ emerged.

### 2. Possible Worlds in Modal Logics  

#### 2.1 Propositional modal logics

**Syntax:** the languages of PML are those of a standard propositional logic to which we add two new sentential operators ‘\( \Box \)’ and ‘\( \Diamond \)’ which function syntactically like ‘\( - \)’.

The propositional logic in *Principia Mathematica* (1913) features the truth-functional connective ‘\( \supset \)’, where \( P \supset Q \) is equivalent to \( \neg (P \& \neg Q) \). And ‘\( P \supset Q \)’ is taken to symbolize ‘If \( P \) then \( Q \)’.

Dissatisfied with this characterization of ‘If... then...’ constructions, C. I. Lewis (1918) introduced the sentential operator ‘\( \Box \)’, with the informal elucidation that it captures the English ‘It is necessary that...’ (and ‘\( \Diamond \)’ ‘It is possible that...’). Once informally understood as absolute or logical necessity, we can define a strict conditional, ‘\( \rightarrow \)’, which is the *necessitation* of \( \supset \).

With that conception of necessity comes the natural endorsement of the following axiom:

\[
(K) \quad (\Box(A \supset C)) \supset (\Box A \supset \Box C)
\]

Various logical systems are defined by adding combinations of different axioms—most notably:

- (T) \( \Box A \supset A \)
- (B) \( A \supset \Box \Diamond A \)
- (S4) \( \Box A \supset \Box \Box A \)
- (S5) \( \Diamond A \supset \Box \Diamond A \)
Adding only (T) and (K) leads to the system $M$. Adding (S4) leads to $S4$. And adding (S5) to $S5$.

Lewis’s work was concerned with showing which formulas can be derived as theorems in which axiom systems.

Pre-theoretical intuitions about absolute necessity leave undetermined how strong a system is required to capture the concept: $M$, $S4$, or $S5$? Uncertainty as to how such questions should so much as be answered was one of the reasons modal logic did not immediately take off.

2.2 Quantified modal logics

Syntax: Just as in non-modal quantificational logic, we have wffs like ‘$\forall x \, Fx$’ and ‘$\neg \exists x \, Fx$’, in QML we have ‘$\Box \exists x \, Fx$’, ‘$\forall x \, \lozenge Fx$’, ‘$\neg \exists x \, \Box Fx$’... Crucially, constructions of QML include occurrences of the modal operators that have open sentences in their scope.

(2) The number of planets (in our solar system) is necessarily greater than 7.

Sentences like (2) give rise to two different readings. On the former, a modal property is attributed to a proposition (this is the de dicto reading). On the latter, a modal property is attributed to an entity (this is de re reading).

We can regiment this ambiguity as follows. Where $N = \text{‘numbers the planets (in our solar system)’}$:

(2’) $\Box \exists x \, (N \, x \, \& \, \forall y \, (Ny \, \rightarrow \, x = y) \, \& \, G \, (x, \, 7))$ De dicto

(2’’) $\exists x \, (N \, x \, \& \, \forall y \, (Ny \, \rightarrow \, x = y) \, \& \, \Box G \, (x, \, 7))$ De re

In short, a formula with modal operators is de re iff it contains a modal operator $M$ which has within its scope a singular term, a free variable, or a variable bound from outside $M$’s scope. A formula is de dicto if it is not de re.

(3) Necessarily, all sparrows are birds. $\Box \forall x \, (Sx \, \rightarrow \, Bx)$ De dicto

(4) All sparrows are necessarily birds. $\forall x \, (Sx \, \rightarrow \, \Box Bx)$ De re

2.3 Simple PWS for QML

We started with PML, expanded it to QML, and now we want to interpret the language. We need a specification of models for QML.

Remember how to define validity in PL using truth tables? A valid formula is one where every row is true. Well, truth tables can’t be used to provide an account of validity in modal logics because, as we saw, ‘Necessary, $p$’ is not extensional.

Kripke’s (1959; 1963) work is the received gold standard. But what I present here is very basic. The simple PW semantic interpretation of QML can be given, first, in its pure, mathematical form.
Pure semantics:

An ‘SQML-model’ is any sequence $< I, D, T >$ where $I$ is a non-empty set (of ‘worlds’), $D$ is a non-empty set (‘domain’), and $T$ is a function (‘interpretation function’) such that:

- $T(\alpha)$ is some member of $D$ for each constant (i.e. name) $\alpha$;
- $T(\Pi^n)$ is a set of $n+1$-tuples of the form $<u_1, u_2 \ldots u_n, w>$, where $u_1, u_2 \ldots u_n$ are members of $D$ and $w$ of $I$, for each $n$-place predicate $\Pi^n$.

We take the interpretations familiar from non-modal predicate logic, and relativize the interpretation of predicates to ‘worlds’. That is, $T(\Pi^n)$ tells us which $n$-tuples of individuals satisfy which predicates at which ‘worlds’.

The interpretation of a one-place predicate in non-modal predicate logic is a set of members of $D$. Now it is a set of ordered pairs.

A valuation function for an interpretation $T$ extends that interpretation to formulas in line with the meanings of constants, predicates, connectives, etc.

Given an SQML-model $M (<I, D, T>)$ and variable assignment $g$, the unique valuation function $V_{M,g}$ is that which assigns 0 or 1 to each wff at each $w$ in $I$, such that:

For any terms $\alpha, \beta$:
- $V_{M,g}(\alpha = \beta, w) = 1$ iff $[[\alpha]]_{M,g} = [[\beta]]_{M,g}$
  - For any $n$-place predicate, $\Pi^n$, and any terms $\alpha_1 \ldots \alpha_n$:
- $V_{M,g}(\Pi \alpha_1 \ldots \alpha_n, w) = 1$ iff $<[\alpha_1]_{M,g}, \ldots [\alpha_n]_{M,g}, w>$ is a member of $T(\Pi^n)$

We can then define recursive rules for interpreting connectives, as well as ‘$\forall$’ and our ‘$\Box$’:

For formula $\phi$:
- $V_{M,g}(\Box \phi, w) = 1$ for every $v$ which is a member of $I$: $V_{M,g}(\phi, v) = 1$
- For formula $\phi$ and variable $\alpha$:
- $V_{M,g}(\forall \alpha \phi, w) = 1$ for every $u$ which is a member of $D$: $V_{M,g}^u(\phi, w) = 1$

$\phi$ is SQML-valid iff $\phi$ is true at every world and every assignment for every SQML-model.

Two things to observe...

(i) This treatment of the modal operators effectively views them as quantifiers over ‘worlds’. So it turns out that the failures of extensionality mentioned earlier are not irremediable failures.
This is all purely mathematical. We’ve not said anything about what ‘worlds’ are, or how this framework might have application to the properly modal notions circumscribed in §1.

Instead of a set of worlds, we could take a model to be a set of moments of time. We might define an operator $H$ as expressing ‘It has always been the case that...’ and take $P$ as its dual, expressing ‘It was at some time the case that...’. This is temporal logic. (It turns out that an axiom like $PP\phi$ iff $P\phi$, which looks intuitive enough, requires the structure of time to be transitive and dense.)

Applied PWs semantics:

If the modal operators can be correctly interpreted as quantifiers over the indices of some or other frame [namely, possible worlds and possible individuals], then we have found out where to look for illumination about controversial axioms. If not, not. To apply the results, you must incur a commitment to some substantive analysis of modality (Lewis 1986: 19).

Someone who buys into the PWS for modal logic will say that the ‘worlds’ of $\mathcal{M}$ are not merely to ‘be regarded as’ possible worlds. There is one intended interpretation of the modal operators which is such that $\mathcal{M}$ is in fact the set of all the possible worlds (circumscribed in §1), $\emptyset$ the set of the possible individuals, and the referents and intensions assigned by $\mathcal{S}$ the ones which the names and predicates of the language in fact have. $\phi$ is true at $w$ iff $\phi$ is true$_m$ at $w$, etc.

PWS, so applied, involves quantification over sets and sequences whose members are possible worlds (and possibilia which exist ‘at’ those worlds).

So long as some coherent and theoretically respectable account of the metaphysics and epistemology of possible worlds is available, then, the existence of a domain of PWs and of individuals which exist at those worlds should be accommodated. We should be realists.

2.4 Modalism

Can we be realists about modal facts, and support the utility of QML, without being committed to PWs and possible individuals? Modalism is the view that the proper expression of modal thought and talk in natural language will feature primitive modal operators, not to be understood in terms of quantification over possible worlds and possible individuals.

The modalist needn’t deny that there is something illuminating about the PWs model theory, but she will take commitment to an ontology of PWs and possible individuals as too high a price to pay. In adopting this strategy, she will also lose any advantages of honest PWs talk—its analysis of counterfactuals, intensions, supervenience, etc. Worse, while she may be able to formalize the
meaning of sentences like ‘Possibly, dinosaurs could still have existed today’, there are sentences she has a lot of trouble with.

(5) In the event of a fire, there are three ways for you to leave the building.

(6) I could have been taller than you are.

It turns out that accommodating numerical quantification over ‘ways’ like (5), and modal comparatives like (6), require introducing a range of dedicated resources which increase the primitives of the language. These aren’t just fringe uses of modal notions. It might be of great interest to know (5), and of scientific interest to know that there are two ways a physical system might evolve from an initial state. See Ch. 4 of J. Melia, *Modality* (2003) for discussion.

2.5 Controversial formulas

The ‘simple’ modal logic above generates the following controversial formula as a theorem.

\[ \Box \exists x Fx \rightarrow \exists x \Box Fx \]

This says that if it is possible for something to be \( F \), then there is something which is possibly \( F \). Maybe you can hear already why that might be controversial. It also leads us to the following schema:

\[ \Box \forall \alpha \Box \exists \beta (\alpha = \beta) \]

\( \Box \) says that necessarily everything is necessarily identical with something. Or, to put it less formally, necessarily everything necessarily exists. This view is known as necessitism.

You might object that it is contingent which things exist. There could have been things—ghosts and talking donkeys—which don’t actually exist. This assumes that what exists varies from world to world. But our SQML-model above took it for granted that there was a constant domain of objects which could be used to populate worlds. Our quantifiers range over the same domain of objects, regardless of which world is at issue. While necessitists accept the conclusion, contingentists revise SQML-models to make room for variable domains.

Most of the positions we’ll be looking at in subsequent lectures will be contingentist. (Strictly speaking, GMR validates the statement of necessitism in QML, but trivialises it.) See Williamson (2013) for a defence of necessitism.
3. Applications / Explanatory Ambitions

The notion [of a PW] will get its content, not from any direct answer to the question, what is a possible world? or from any reduction of that notion to something more basic, but from its role in the explanations of a theory [of rational activity]. Thus it may be that the best philosophical defence that one can give for possible worlds is to use them in the development of substantive theory (Stalnaker 1975: 141).

We’ll explore theoretical applications in detail next week. For now, here is one.

3.1 Intensions

Analyses provided for intensional entities?

Propositions: functions from worlds to truth-values
Properties: functions from worlds to sets of individuals
n-place Relations: functions from worlds to sets of n-tuples of individuals

Functions themselves are extensional creatures, analysable with mere set theory, and are to that extent free from obscurity. Richard Montague’s work generalized these sorts of definitions, giving intensional treatments of verbs, adjectives, and much more.

Sufficiently fine-grained? Equivalence of necessary propositions and triangularity/tri-lateralness...

4. Conclusion

Possible worlds are incorrectly thought of as postulated entities introduced into our theories to explain independently identifiable features of modal discourse. [...] The term ‘possible world’ is just the philosopher’s name for things which we have recognized all along. But, then, it would seem that no theory that refuses to countenance the framework of possible worlds can be a really satisfactory account of modality. Something central to modal thinking will be left out (Loux 1979: 62).

Further reading: